Multiresolution PPM for Broadcasting over Asymmetric Photon Counting Channels

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Abstract—**In this letter, we address the problem of broadcasting to receivers with different bandwidths using pulse-position modulation (PPM). A new multiresolution PPM scheme is developed for such a scenario. We show how to construct the scheme for various settings and analyse its performance in the solar-blind region of the ultraviolet (UV) spectrum, where photon counting is attractive. Our key result is that the proposed scheme can increase information rate compared to time-sharing (TS) by superimposing information.**

Index Terms—**Broadcasting, PPM, superpostion coding, photon counting.**

I. Introduction

RADIO frequency (RF) systems are widely used for wireless data transmission, which is leading to a highly congested RF spectrum. A potential solution to meet the increasing demand for higher data rates is optical wireless (OW). The wide bandwidth of OW, which uses an unlicensed spectrum, makes it a promising candidate for future wireless communication systems.

Fig. 1 shows a diagram of an OW system. A transmitter, usually mounted on a ceiling, transmits data to receivers in its cone of illumination. Such systems might contain terminals with different capabilities ranging from simple wireless sensors to sophisticated portable computers. Optical receivers in each of these terminals are likely to have different bandwidths, and therefore receive data at different rates. In this setting it is desirable to be able to send information to both receivers at rates suited to their available bandwidth.

Scattering in the solar-blind region of the UV spectrum (200 to 280 nm) creates non-line-of-sight channels, which has been widely researched as a means to provide wireless communication. Over short distances, where scattering is not significant, line-of-sight broadcasting in this region with photon counting receivers is potentially attractive, as there is negligible ambient sunlight light [1].

A simple protocol for broadcasting is TS. However, the information-theoretic study in [2] showed that TS is a lower bound on the information rate and more information can be transmitted by superposition coding (SC) [3]. A rich body of research focuses on developing various multiresolution modulation schemes that operate on the principle of SC and analysing their performance [4], [5], [6]. These modulation schemes are designed for receivers with different signal-to-noise ratios. So far, however, there has been little discussion about designing modulation schemes for broadcasting when receivers have different bandwidths. Most work assumes additive white Gaussian noise (AWGN) channels [7], and references therein. However, the negligible ambient sunlight and the sensitive receivers, considered here allow communication systems to operate in a photon counting regime, where Poisson statistics are observed.

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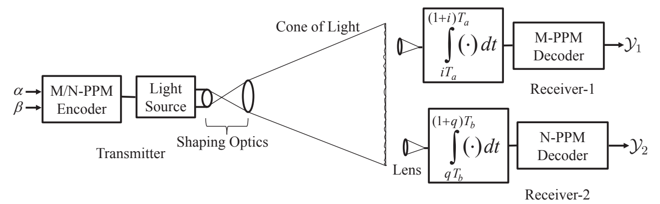

Fig. 1. Schematic of transmitter broadcasting to receivers by optical wireless. $i = 0, 1, \ldots, Q$; $q = 0, 1, \ldots, N$.

Our key contributions are twofold. First, we propose a new PPM scheme for broadcasting over channels with different bandwidths. Second, we analyse its performance in a photon counting regime. The remaining of the paper is organised as follows. In Section II, we present our system model. In Section III, we introduce the proposed scheme, which is then analysed in Section IV. Finally, Section V concludes the paper with a summary and future research directions.

II. System Model

Consider the communication scenario depicted in Fig. 1, where a transmitter sends independent information to two receivers with different bandwidths, i.e., receiver-1 has a higher bandwidth than receiver-2. The transmitter sends photons and receivers count the number of photons that arrive, k, according to a Poisson distribution

$$p(k|\lambda) = e^{-\lambda}\frac{\lambda^k}{k!}, \quad \text{for } k = 0, 1, 2, \ldots \text{ and } \lambda > 0, \tag{1}$$

where λ is the average number of received photons after path loss. We assume a channel model where no background noise is detected, and the only source of error is when no photons are received [8]. Additionally, we assume perfect synchronization between the transmitter and receivers.

III. Proposed Scheme

Here we present the proposed M/N-PPM scheme. Let $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ denote the binary sequences of receiver 1 and 2 at the transmitter, respectively

$$\boldsymbol{\alpha} = (a_0^0, a_0^1, \ldots, a_0^m, \ldots, a_l^0, a_l^1, \ldots, a_l^m),$$
$$\boldsymbol{\beta} = (b_0^0, b_0^1, \ldots, b_0^n, \ldots, b_l^0, b_l^1, \ldots, b_l^n),$$

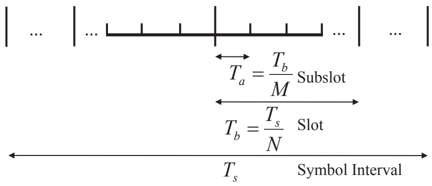

Fig. 2. General structure of an M/N-PPM frame.

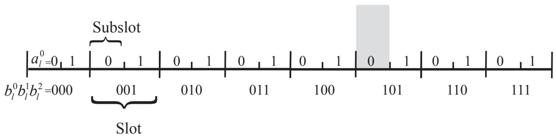

Fig. 3. Multiresolution PPM frame; M=2, and N=8.

$$a_l^j, b_l^j \in \{0,1\}, m = \log_2(M), n = \log_2(N), l = 0, 1, \ldots$$

where a_l^j is the jth bit in a m-tuple, and a similar notation is used to describe the elements in $\boldsymbol{\beta}$.

Fig. 2 shows the general structure of an M/N-PPM frame. In each symbol interval, a PPM encoder maps a block of $\log_2(Q)$ bits to a channel symbol. The PPM symbol interval, T_s, is divided into N time slots of duration T_b, and each slot is further divided into M subslots of duration T_a. Here, M is the number of time slots allocated to receiver-1, and similarly N is the number of time slots allocated to receiver-2. The transmitter sends photons in one of the $Q \triangleq MN$ subslots. Receiver-1 counts the number of photons in each subslot, whereas receiver-2 counts the number of photons in each slot to determine the transmitted symbol.

Encoder: The M/N-PPM encoder at the transmitter maps inputs from receiver 1 and 2 into a codeword

$$\psi : \{0, 1, \cdots, 2^n - 1\} \times \{0, 1, \cdots, 2^m - 1\} \to \mathcal{X}. \quad (2)$$

Decoders: The output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$ have a larger alphabet than the input. This is due to the erasure case, $\mathscr{E}$, that arises when no photons are detected in a symbol interval, which occurs with a probability of $p(0|\lambda) = e^{-\lambda}$;

$$\phi_1 : \mathcal{Y}_1 \to \{0, 1, \cdots, 2^m - 1, \mathscr{E}\}, \quad (3)$$

$$\phi_2 : \mathcal{Y}_2 \to \{0, 1, \cdots, 2^n - 1, \mathscr{E}\}, \quad (4)$$

where ϕ_1 and ϕ_2 are the decoders at receivers 1 and 2, respectively.

Fig. 3 shows an example of transmission. Here, the bandwidth of receiver-1 is twice of that of receiver-2, and the transmitted bits are $a_l^0 = 0$, and $b_l^0 b_l^1 b_l^2 = 101$ to receiver-1 and receiver-2, respectively. In this example, a 2-PPM scheme is used for transmission to receiver-1, and an 8-PPM scheme for receiver-2. The transmitter uses a multiresolution 2/8-PPM frame, and sends a pulse in the depicted subslot in Fig. 3. Receiver-1 counts the number of photons in each of the 16 subslots of the 2/8-PPM frame, while receiver-2 counts the number of photons in each of the 8 slots of the 2/8-PPM frame to determine the transmitted information.

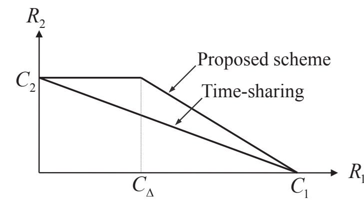

Fig. 4. Capacity region.

IV. Performance Analysis

In this section we analyse the capacity, and signalling efficiency of the proposed M/N-PPM scheme.

A. Capacity

The capacity of an L-PPM scheme in a Poisson channel is [9]

$$C = (1 - e^{-\lambda}) \ln(L) \qquad \text{nats/channel use.} \quad (5)$$

Since the channel is used every $T_a M$ seconds by receiver-1, its capacity is

$$C_1 = \frac{(1 - e^{-\lambda})}{T_a M} \ln(M) \qquad \text{nats/sec.} \quad (6)$$

When using the proposed scheme the channel is used every $T_a Q$ seconds; resulting in the following capacity of receiver-1

$$C_\Delta = \frac{(1 - e^{-\lambda})}{T_a Q} \ln(M) \qquad \text{nats/sec.} \quad (7)$$

Similarly, the capacity of receiver-2 is

$$C_2 = \frac{(1 - e^{-\lambda})}{T_a Q} \ln(N) \qquad \text{nats/sec.} \quad (8)$$

When using TS for broadcasting, a fraction τ of the total broadcasting time is allocated to transmitting to receiver-1 and $(1-\tau)$ to receiver-2. The following rate pairs characterize the achievable rates of this protocol

$$C_{TS} = \bigcup_{\{\tau:\ 0 \le \tau \le 1\}} (R_1 = \tau C_1, R_2 = (1 - \tau) C_2), \quad (9)$$

where R_1 and R_2 are the rates of receiver 1 and 2, respectively. Fig.4 shows the capacity region of this broadcasting protocol. It can be seen that TS can increase the rate of one receiver by reducing the rate of the other.
However, using the proposed scheme superimposed information can be communicated to receiver-1 at a rate of C_Δ. This yields a larger capacity region than TS (Fig.4).

The average received power is given by

$$P = \frac{hv\lambda}{T_a Q} \qquad \text{watts,} \quad (10)$$

where h is Planck's constant and v is the photon frequency. Substituting the value of λ from (10) into (7) and (8), we get the relation between information rate and power for receiver 1 and 2, respectively

$$C_\Delta = \frac{1 - e^{-\frac{P T_a Q}{hv}}}{T_a Q} \ln(M) \qquad \text{nats/sec,} \quad (11)$$

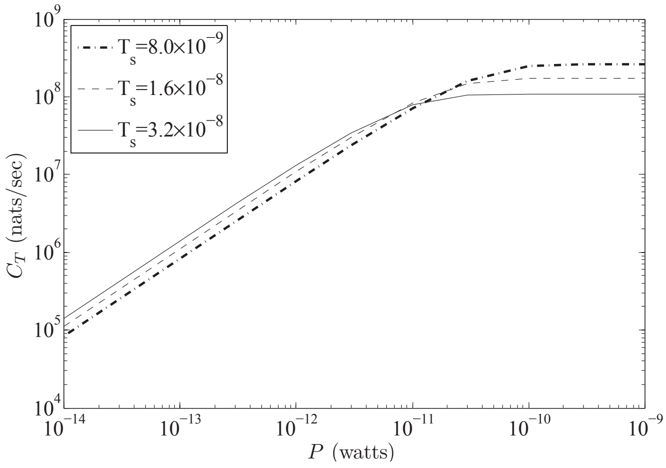

Fig. 5. Capacity versus power (P); $M = 2$ $subslots$, $T_a = 1$ ns, and wavelength = 240 nm.

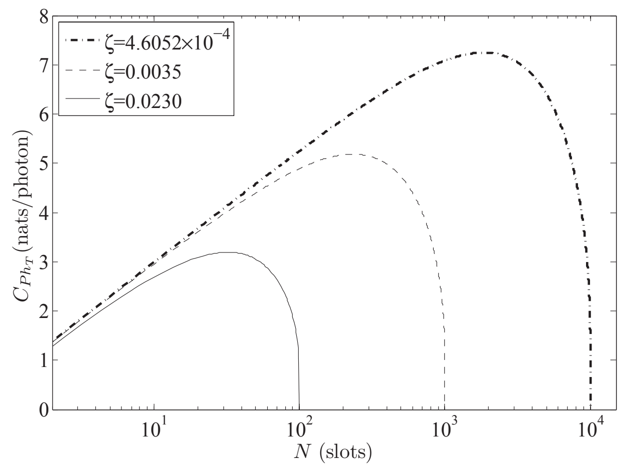

Fig. 6. Total signalling efficiency versus number of time slots (N); $M = 2$ $subslots$, and $\zeta \triangleq C_2 T_a$ $nats/subslot$.

$$C_2 = \frac{1 - e^{-\frac{PT_aQ}{hv}}}{T_aQ} \ln(N) \qquad \text{nats/sec.} \qquad (12)$$

And the total capacity is $C_T = C_\Delta + C_2 = \frac{1-e^{-\frac{PT_aQ}{hv}}}{T_aQ} \ln(Q)$ $nats/sec$. For a fixed value of Q, P, and v the supremum of C_T occurs as $T_a \to 0^+$

$$\lim_{T_a \to 0^+} C_T = \lim_{T_a \to 0^+} \frac{1 - e^{-\frac{PT_aQ}{hv}}}{T_aQ} \ln(Q) \qquad (13)$$

$$= \frac{P}{hv} \ln(Q) \qquad \text{nats/sec.} \qquad (14)$$

This quantity establishes an upper bound on the information rate of the proposed scheme. Fig. 5 shows the performance of the proposed M/N-PPM scheme versus power. Here, the value of T_a is set to 1 ns, since systems with speeds in the $Gbits/sec$ range operate near this value of T_a. As shown the information rate increases with power, and saturates as the system approaches its capacity, since the probability of an erasure decays exponentially with power.

B. Signalling Efficiency

The signalling efficiency, defined as the amount of information conveyed by a photon $C_{Ph} = \frac{C}{\lambda}$, of receiver-2 is

$$C_{Ph_2} = \frac{(1 - e^{-\lambda})}{\lambda} \ln(N) \qquad \text{nats/photon.} \qquad (15)$$

Substituting the value of λ from (8) in (15) and rearranging we get

$$C_{Ph_2} = \frac{C_2 T_a Q}{\ln(1 - \frac{C_2 T_a Q}{\ln(N)})^{-1}} \qquad \text{nats/photon.} \qquad (16)$$

Since $\log_2(M)$ bits are conveyed to receiver-1 for every $\log_2(N)$ bits conveyed to receiver-2 via the proposed scheme, it can be shown that the signalling efficiency of receiver-1 is

$$C_{Ph_1} = C_{Ph_2} \log_N(M) \qquad \text{nats/photon.} \qquad (17)$$

Fig. 6 shows the total signalling efficiency, $C_{Ph_T} = C_{Ph_1} + C_{Ph_2}$, versus N. It can been seen that there exists an optimum value of N at which the signalling efficiency is maximised. This optimum is unique for each value of $\zeta \triangleq C_2 T_a$; defined as the amount of information per subslot conveyed to receiver-2.

V. Conclusion

A new PPM scheme has been presented for broadcasting over asymmetric photon counting channels. It is shown that, information can be superimposed by the proposed scheme and therefore can increase the information rate compared to the traditional TS approach. It is further shown that there exists an optimum operating point at which signalling efficiency of the proposed scheme is maximised. Future work will examine the performance of the M/N-PPM scheme when erasure codes are employed, and include a practical demonstration.

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