Anti-plane dynamic shear of three-layered asymmetric high-contrast plate

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Abstract
Anti-plane dynamic shear of asymmetric three-layered laminates is analysed.One type of high contrast is considered, including composite structure with at least one stiff (thick or thin) layer. It is shown the value of the cut-off frequency, corresponding to the lowest vibration mode for three-layered asymmetric plate with traction free faces tends to zero. Numerical data illustrating comparisons of exact and asymptotic results are presented.

Antiplane shear of three- layered asymmetric solids

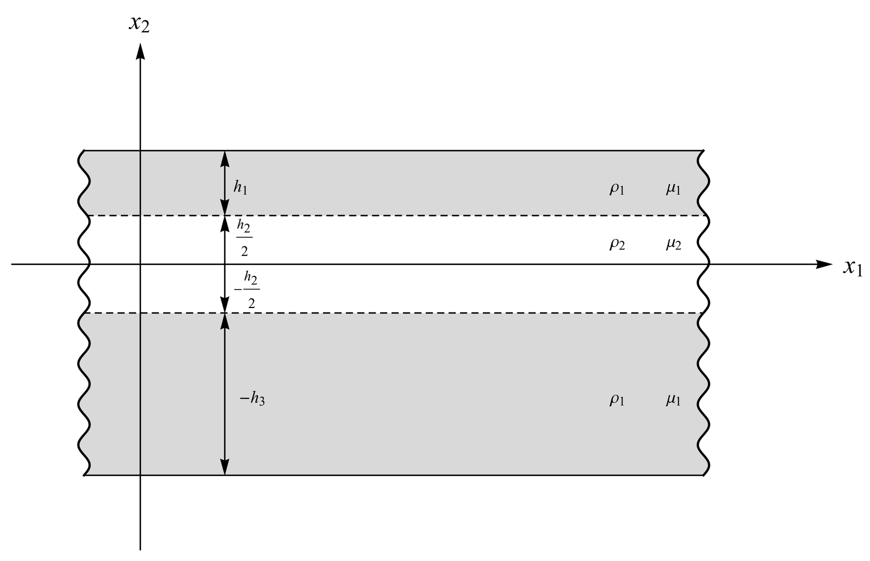

Figure 1: Three-layered laminate

Equations of motion

The equations of motion can be written as

$$(\lambda+\mu)U^L_{k,kj}+\mu\triangle U^L_j=\rho U^L_{j,tt} \quad (1)$$

where L= 1,2,3 indicates the layer of the laminate , λ and μ are Lame parameters , and ρ is mass density.

Continuity conditions

The continuity conditions for the three-layered asymmetric plate are given by

$$U_3^{(1)}=U_3^{(2)} \qquad at \qquad x_2=h_2/2 \quad (2)$$
$$\sigma_{23}^{(1)}=\sigma_{23}^{(2)} \qquad at \qquad x_2=h_2/2 \quad (3)$$

$$U_3^{(2)}=U_3^{(3)} \qquad at \qquad x_2=-h_2/2 \quad (4)$$
$$\sigma_{23}^{(2)}=\sigma_{23}^{(3)} \qquad at \qquad x_2=-h_2/2 \quad (5)$$

Boundary conditions

We also impose the traction-free boundary conditions for three-layered laminate

$$\sigma_{23}^{(1)}=0 \qquad at \qquad x_2=((h_2/2)+h_1) \quad (6)$$

$$\sigma_{23}^{(3)}=0 \qquad at \qquad x_2=(-(h_2/2)-h_3) \quad (7)$$

Dispersion relation

The dispersion relation associated with three-layered asymmetric plate is given by

$$\mu\alpha_1\alpha_2\tanh(h\alpha_1)+\mu^2{\alpha_2}^2\tanh(\alpha_2)+\mu\alpha_1\alpha_2\tanh(h^*\alpha_1)+{\alpha_1}^2\tanh(h^*\alpha_1)\tanh(\alpha_2)\tanh(h\alpha_1)=0 \quad (8)$$

where

$$\alpha_1=\sqrt{K^2-\frac{\mu}{\rho}\Omega^2}, \qquad \alpha_2=\sqrt{K^2-\Omega^2}. \quad (9)$$

Non-dimensional frequency Ω and wavenumber K are introduced as

$$\Omega=\frac{\omega h_2}{c_{22}} \qquad K=kh_2 \quad (10)$$

Contrast parameters

$$h=\frac{h_1}{h_2} \qquad h^*=\frac{h_3}{h_2} \qquad \mu=\frac{\mu_2}{\mu_1} \qquad \rho=\frac{\rho_2}{\rho_1} \qquad c_{2i}=\sqrt{\frac{\mu_i}{\rho_i}} \qquad i=1,2 \quad (11)$$

Dispersion curves computed from (8) are plotted in Figure 2 for non-contrast and contrast cases. Due to a high contrast in density and stiffness of the layers, the first cut-off is close to zero.

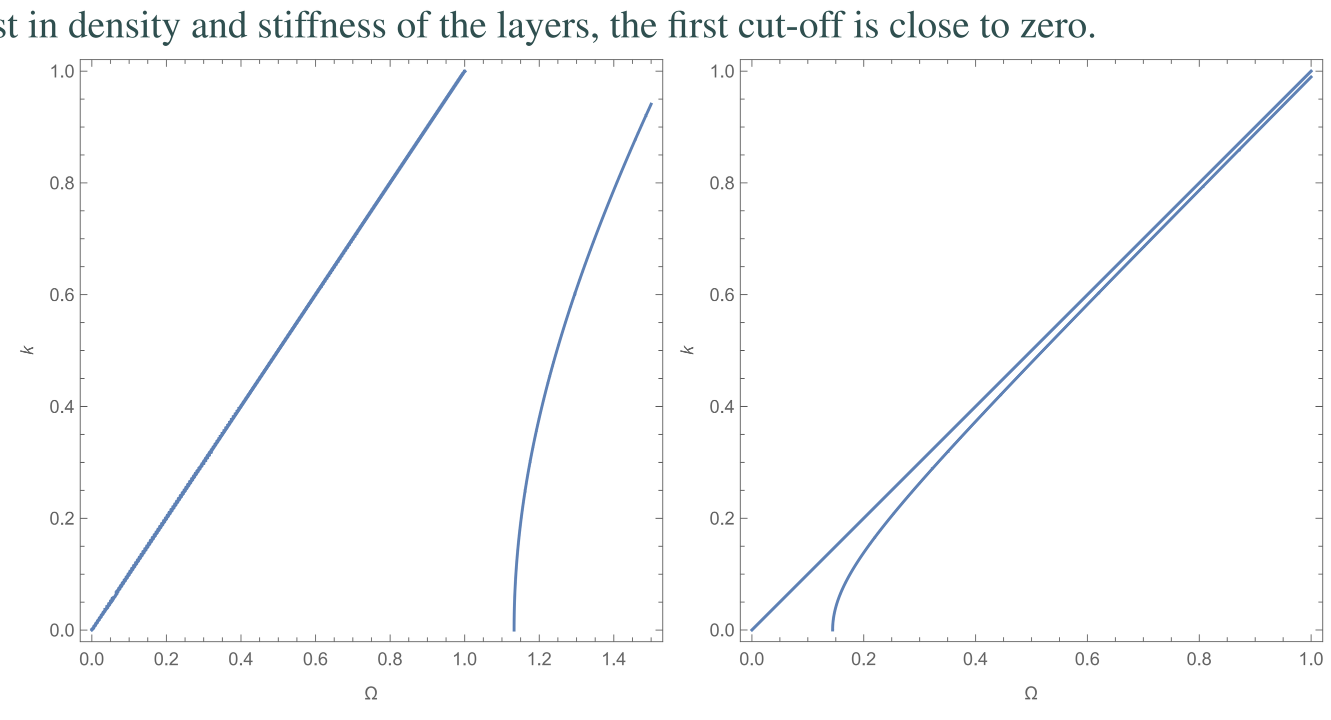

Figure 2: Dispersion curves for an antiplane shear of tree-layerd asymmetric laminate with contrast (b) and without contrast(a)

We study one of the setups of the contrast, mentioned in [1],in particular

$$\mu\ll 1, h\sim 1, \rho\sim\mu$$

Lowest cut-off and polynomial dispersion relation

The frequency equation for three-layered asymmetric plate is given by

$$\sqrt{\mu\rho}\left(\tan\left(h\sqrt{\frac{\mu}{\rho}}\Omega\right)+\tan\left(h^*\sqrt{\frac{\mu}{\rho}}\Omega\right)\right)+\mu\rho\tan(\Omega)-\tan\left(h\sqrt{\frac{\mu}{\rho}}\Omega\right)\tan(\Omega)\tan\left(h^*\sqrt{\frac{\mu}{\rho}}\Omega\right)=0 \quad (12)$$

This is used to demonstrate that for the global low- frequency regime at leading order we have

$$\Omega\approx\sqrt{\frac{\mu\rho(h+h^*+\rho)}{hh^*\mu}} \quad (13)$$

Long- wave low- frequency approximation

Now, we expand (8)into asymptotic series, resulting in the polynomial dispersion relation

$$\gamma_1K^2+\gamma_2K^4+\gamma_3K^2\Omega^2+\gamma_4\Omega^2+\gamma_5\Omega^4+\gamma_6K^4\Omega^2+\gamma_7K^8+\gamma_8K^2\Omega^4+\gamma_9K^6\Omega^2+\gamma_{10}K^{10}+...=0 \quad (14)$$

where γ_i are coefficients depending on parameters ρ,μ,h,h^*.
i=1,2,3,..

An example of a shortened polynomial dispersion relation

For two modes we get the shortent dispersion relation,

$$K^4\Omega^2\left(\frac{2\mu^2\left(h^5+h^{*5}\right)}{5\rho}\right)-K^2\Omega^4\left(\frac{2\mu^3\left(h^5+h^{*5}\right)}{5\rho^2}\right)+K^4\left(\frac{1}{3}\mu(h^3+h^{*3})-hh^*\right)+K^2\Omega^2\left(\frac{2hh^*\mu}{\rho}\right)+K^2\left(\mu(-h-h^*)\right)-\Omega^4\left(\frac{hh^*\mu^2}{\rho^2}\right)+\Omega^2\left(\frac{\mu^2(h+h^*)}{\rho}\right)=0 \quad (15)$$

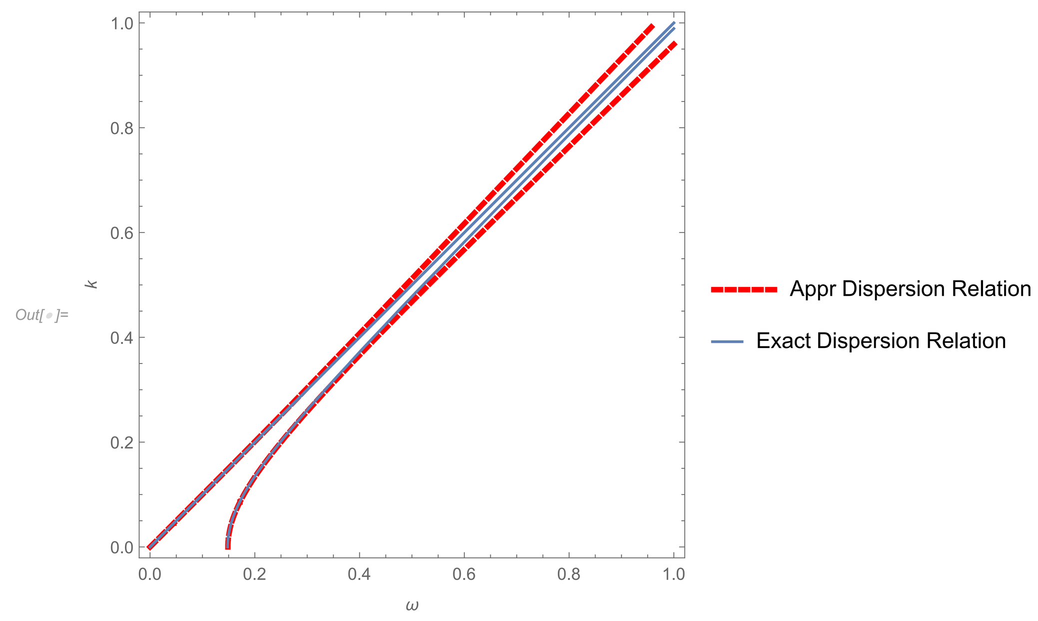

Figure 3: Exact dispersion equation (8) (green solid line) and asymptotic dispersion (15) (red dashed line)

Asymptotic variation across thickness

For leading order displacements, we get

$$U_1=\frac{\sqrt{\mu}\sqrt{K_*^2-\frac{\Omega_*^2}{\rho_*^2}}}{h(K_*^2-\Omega_*^2)} \quad (16)$$

$$U_2=\frac{\sqrt{\mu}\sqrt{\frac{K_*^2\rho_*^2-\Omega_*^2}{\rho_*^2}}(2h_2+hh_2K_*^2x_2-hh_2\Omega_*^2+2hx_2\Omega_*^2)}{2hh_2(K_*^2-\Omega_*^2)} \quad (17)$$

$$U_3=\frac{\sqrt{\mu}\sqrt{\frac{K_*^2\rho_*^2-\Omega_*^2}{\rho_*^2}}(-1-K_*^2+h\Omega_*^2)}{h(K_*^2-\Omega_*^2)} \quad (18)$$

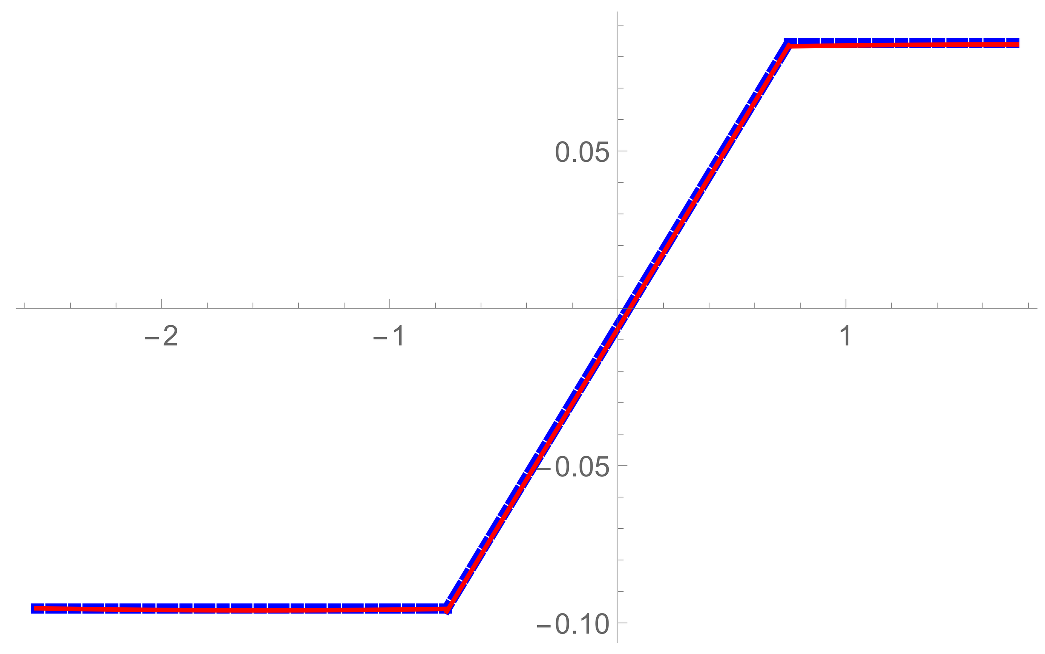

Figure 4: Normalized displacement $U^{(i)}$ i=1,2,3. Exact solution(red solid line) and asypmtotic one (blue dashed line))

Conclusions

We demonstrated that the three layered asymmetric laminates the first cut off frequency becomes asymptotically small in case of high contrast properties of the layers. Approximate relations are derived for the long wave low frequency regime, demonstrating good agreement with the numerical implementation of the exact solutions.

References

[1] J. Kaplunov, D.A. Prikazchikov, L.A. Prikazchikova *"Dispersion of elastic waves in a strongly inhomogeneous three-layered plate"* Int. J. Solids Struct., 113 (2017), pp. 169-179

[2] Y. E. Aydin, B. Erbas, J.Kaplunov, and L. Prikazchikova *"Asymptotic analysis of an antiplane dynamic problem for a three-layered strongly inhomogeneous laminate"*